



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS:

Lecture 12: Footings under Dynamic Loads: Numerical on Analytical Approaches

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ERD of Foundations - Modules

1. Introduction
- 2. Shallow Foundations**
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4. Well Foundations
5. SSI for Deep Foundations

Module - 2

Shallow Foundations



Module -2: Chapters

1. General Requirements
2. Bearing Capacity and Settlement
3. Combined Footings
4. Raft Foundations
5. Footings Subjected to Dynamic Loads

Dynamic Analysis

All analytical approaches are based on the assumption that soil rupture under transient loads occurs along a static rupture surface

- Transient Vertical (rotation) load (Triandafilidis, 1965)
- Transient Vertical (reciprocating) load (Wallace, 1961)
- Transient Horizontal (reciprocating) Load (Chummar, 1965)
- Numerical Examples

Chummar's Solution

Chummar (1965) presented a solution for dynamic response of a strip footing supported by $c - \phi$ soil and subjected to horizontal transient load.

Assumptions:

1. The failure of the footing occurs with the application of horizontal dynamic load acting at a certain height above the base of the footing. .
2. The resulting motion in the footing is of a rotatory nature. The failure surface is a logarithmic spiral with its centre on the base corner of the footing, which is also the centre of rotation (Fig. next slide).
3. The rotating soil mass is considered to be a rigid body rotating about a fixed axis.
4. The soil exhibits rigid plastic, stress-strain characteristics.

Chummar's Solution (Continued...)

The static bearing capacity of the footing is calculated by assuming that the footing fails when acted upon by a vertical static load, which causes rotation of the logarithmic spiral failure. The ultimate static bearing capacity q_u is given by

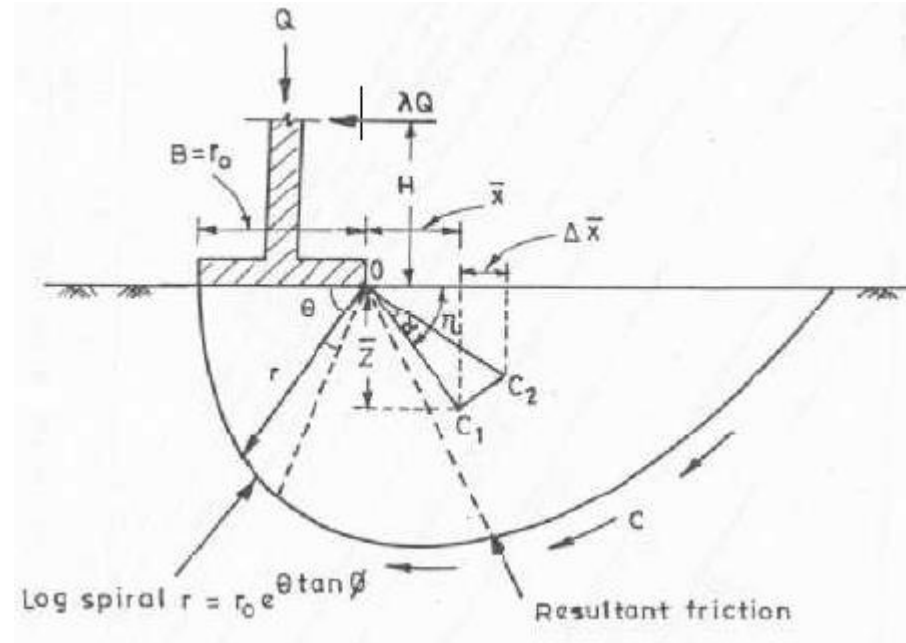
$$q_u = cN_c + \frac{1}{2}\gamma BN_\gamma \quad (1)$$

Where,

c = Cohesion

γ = Unit weight of the soil

B = Footing width and equal to the initial radius of spiral



Chummar's Solution (Continued...)

Considering moment of the forces about O, the centre of rotation:

1 Moment due to cohesion c ,

$$M_{RC} = \frac{cB^2}{2 \tan \phi} (e^{2\pi \tan \phi} - 1) = \psi c B^2$$

2 Moment due to weight W of soil wedge,

$$M_{RW} = \gamma \frac{B^3 \tan \phi (e^{3\pi \tan \phi} + 1)}{9 \tan^2 \phi + 1} = \epsilon \gamma B^3$$

Moment of q_u about O = $q_u \frac{B^2}{2} = M_{RC} + M_{RW}$

$$q_u = \frac{c}{\tan \phi} (e^{2\pi \tan \phi} - 1) + \frac{2\gamma B \tan \phi (e^{3\pi \tan \phi} + 1)}{9 \tan^2 \phi + 1} \quad (2)$$

$$\text{Combining equation 1 and 2, } N_\gamma = \frac{4 \tan \phi (e^{3\pi \tan \phi} + 1)}{9 \tan^2 \phi + 1} \quad N_c = \frac{e^{2\pi \tan \phi} - 1}{\tan \phi}$$

Chummar's Solution (Continued...)

With a suitable factor of safety F , the static vertical force on the foundation per unit length can be given as

$$Q = \frac{B}{F} (cN_c + \frac{1}{2}\gamma BN_\gamma)$$

The variation of dynamic force considered in the analysis is shown in Fig.

$$Q_{d(max)} = \lambda Q$$

$Q_{d(max)}$ = Maximum value of horizontal transient load per unit length acting at height H above base of the footing

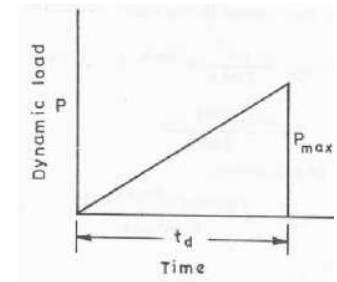
λ = Over load factor

3 Moment due to the vertical force Q

$$M_1 = 0.5QB$$

4 Moment due to the horizontal force Q_d at any time t

$$M_2 = Q_d H = Q_{d(max)} H t / t_d = M_{d(max)} t / t_d$$



Chummar's Solution (Continued...)

5 Moment due to the vertical force Q

$$M_3 = W \Delta \bar{x}$$

$$\text{Where, } W = \frac{\gamma B^2 (e^{3\pi \tan \phi + 1})}{3(\sqrt{9 \tan^2 \phi + 1})} \quad \text{and} \quad \Delta \bar{x} = R \cos(\eta - \alpha) - R \cos(\eta)$$

And $R = OC_1$ when α is small then

$$\Delta \bar{x} = (R \sin \eta) \alpha = \bar{z} \alpha$$

$$R = \sqrt{(\bar{x}^2 + \bar{z}^2)}$$

$$\text{Where, } \bar{x} = \frac{-4B \tan^2 \phi (e^{3\pi \tan \phi + 1})}{(9 \tan^2 \phi + 1)(e^{2\pi \tan \phi - 1})} \quad \bar{z} = \frac{4B \tan \phi (e^{3\pi \tan \phi + 1})}{3(\sqrt{9 \tan^2 \phi + 1})(e^{2\pi \tan \phi - 1})}$$

$$M_3 = \gamma \beta B^2 \bar{z} \alpha$$

$$\text{Where, } \beta = \frac{(e^{3\pi \tan \phi + 1})}{3(\sqrt{9 \tan^2 \phi + 1})}$$

Chummar's Solution (Continued...)

5 Moment due to inertia force of soil wedge:

$$M_4 = \left(\frac{d^2 \alpha}{dt^2} \right) J$$

Where, J = mass moment of inertia of the soil wedge = $\left[\frac{\gamma B^4}{16g \tan \phi} \right] (e^{4\pi \tan \phi} - 1)$

$$M_4 = \frac{\mu_c \gamma B^4}{g} \left(\frac{d^2 \alpha}{dt^2} \right) \quad \text{where, } \mu_c = \frac{(e^{4\pi \tan \phi} - 1)}{16 \tan \phi}$$

Using moment equilibrium: Equation of motion

$$M_1 + M_2 = M_{RC} + M_{RW} + M_3 + M_4$$

Substituting all five moment in above equation

$$\left(\frac{d^2 \alpha}{dt^2} \right) + k^2 \alpha = A \left[\left(\frac{M_{d(max)t}}{t_d} \right) + \frac{1}{2} QB - E \right]$$

$$\text{Where, } k = \sqrt{\frac{g\beta \sin \eta}{\mu_c B}} \quad A = \frac{g}{(\gamma B^4 \mu_c)} \quad E = \psi c B^2 + \epsilon \gamma B^3$$

Chummar's Solution (Continued...)

Solution of differential equation motion with proper boundary condition yields the following results:

$$\text{For } t \leq t_d, \quad \alpha = \frac{A}{k^2} \left(E - \frac{1}{2} QB \right) \cos(kt) - \frac{A}{k^3} \cdot \frac{M_{d(max)}}{t_d} \sin(kt) + \frac{A}{k^2} \left(\frac{M_{d(max)}t}{t_d} + \frac{1}{2} QB - E \right)$$

$$\text{For } t > t_d, \quad \alpha = \frac{1}{k} [G_1 k \cos(kt_d) - G_2 k \sin(kt_d)] \cos(kt) + \frac{1}{k} [G_1 k \sin(kt_d) - G_2 k \cos(kt_d)] \sin(kt) + \frac{A}{k^2} \left(\frac{1}{2} QB - E \right)$$

$$\text{Where, } G_1 = \frac{A}{k^2} \left(E - \frac{1}{2} QB \right) \cos(kt_d) - \frac{A}{k^3} \cdot \frac{M_{d(max)}}{t_d} \sin(kt_d) + \left(\frac{A M_{d(max)}}{k^2} \right)$$

$$G_2 = -\frac{A}{k} \left(E - \frac{1}{2} QB \right) \sin(kt_d) - \frac{A}{k^2} \cdot \frac{M_{d(max)}}{t_d} \cos(kt_d) + \left(\frac{A M_{d(max)}}{k^2 t_d} \right)$$

Numerical Example-1

A 1.5 m wide strip foundation is subjected to a vertical transient stress pulse which can be given as $q_d = 650e^{-10t}$ kN/m². The soil supporting the foundation is saturated clay with $C_u = 60$ kN/m². The unit weight of soil is 19 kN/m³. Determine the maximum angular rotation of the footing might undergo.

Solution

$$q_u = 5.54 c_u = 5.54 \times 60 = 332.4 \text{ kN/m}^2$$

$$\lambda = 650/332.4 = 1.955$$

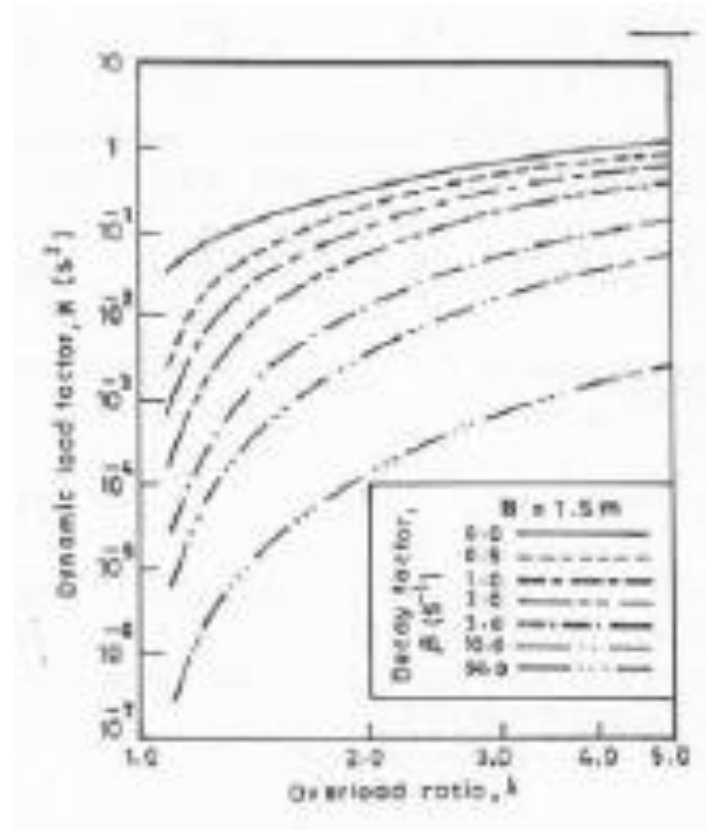
$$\text{for } \lambda = 1.955 \text{ and } \beta = 10 \text{ s}^{-1}$$

$$\frac{W}{0.68 g q_u} \theta_{max} = 0.00298$$

$$W = 0.31 \pi \gamma B^2 = 0.31 \pi * 19 * 1.5^2 = 41.61 \text{ kN}$$

$$\theta_{max} = 0.158 \text{ rad} = 9.1^\circ$$

Numerical Example-1 (Continue)



Numerical Example-2

A 2.5 m wide continuous footing located at 1.5m below the ground surface is subjected to a vertical transient load ($q_{d(\max)} = 3000 \text{ kN/m}^2$, $t_d = 0.3\text{s}$). The properties of the soil are $\gamma = 18 \text{ kN/m}^3$, $\phi = 30^\circ$ and $c = 50 \text{ kN/m}^2$. Calculate the maximum vertical movement of the foundation.

Solution

$$q_u = cN_c + \gamma D_f N_q + \frac{1}{2} \gamma B N_\gamma$$

$$q_u = 50N_c + 18 * 1.5 * N_q + \frac{1}{2} * 18 * 2.5 * N_\gamma$$

$$q_u = 50N_c + 27N_q + 22.5N_\gamma$$

The computations of q_u are done for $\phi = 30^\circ$ and different values of K by taking N_c , N_q and N_γ factors from the table as given in next slide.

Numerical Example-2 (Continued...)

Table 6.5: Computations of q_u for Different Values of k

k	q_u (kN/m ²)
- 0.45	5765
- 0.40	4553
- 0.35	3910
- 0.30	3533
- 0.25	3304
- 0.20	2512
- 0.15	2204
- 0.10	3070
- 0.05	3080
- 0.00	3118
+ 0.05	3181

ϕ (deg)	K	N_Y	N_c	N_q	N_l	N_R	$\sqrt{\frac{N_R}{N_l}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
30	- 0.45	19.3095	80.8644	47.6872	0.1064	9.3123	9.3540
	- 0.40	19.1315	62.4470	37.0539	0.1267	9.0899	8.4705
	- 0.35	19.3718	52.5548	31.3426	0.1506	9.0494	7.7518
	- 0.30	19.940	46.6067	27.9084	0.1787	9.1446	7.1533
	- 0.25	20.1887	42.8208	25.7226	0.2116	9.3473	6.6458
	- 0.20	21.9566	40.3597	24.3017	0.2500	9.6392	6.2095
	- 0.15	23.3512	38.7778	23.3884	0.2944	10.0081	5.8303
	- 0.10	24.9984	37.8159	22.8330	0.3456	10.4452	5.4979
	- 0.05	26.8993	37.3127	22.5425	0.4041	10.9441	5.2044
	0.00	29.0580	37.1624	22.4558	0.4706	11.4998	4.9436
	+ 0.05	31.4810	37.2926	22.5309	0.5457	12.1084	4.7107

Numerical Example-2 (Continued...)

The minimum value of q_u is obtained as 2204 kN/m² at $K = -0.15$.

For $K = -0.15$ and $\phi = 30^\circ$ from the table in previous slide.

$$N_R = 10.0081 \text{ and } \sqrt{\frac{N_R}{N_I}} = 5.8303$$

$$\frac{N_R \gamma}{q_u} = \frac{10.0081 * 18}{2204} = 0.08174$$

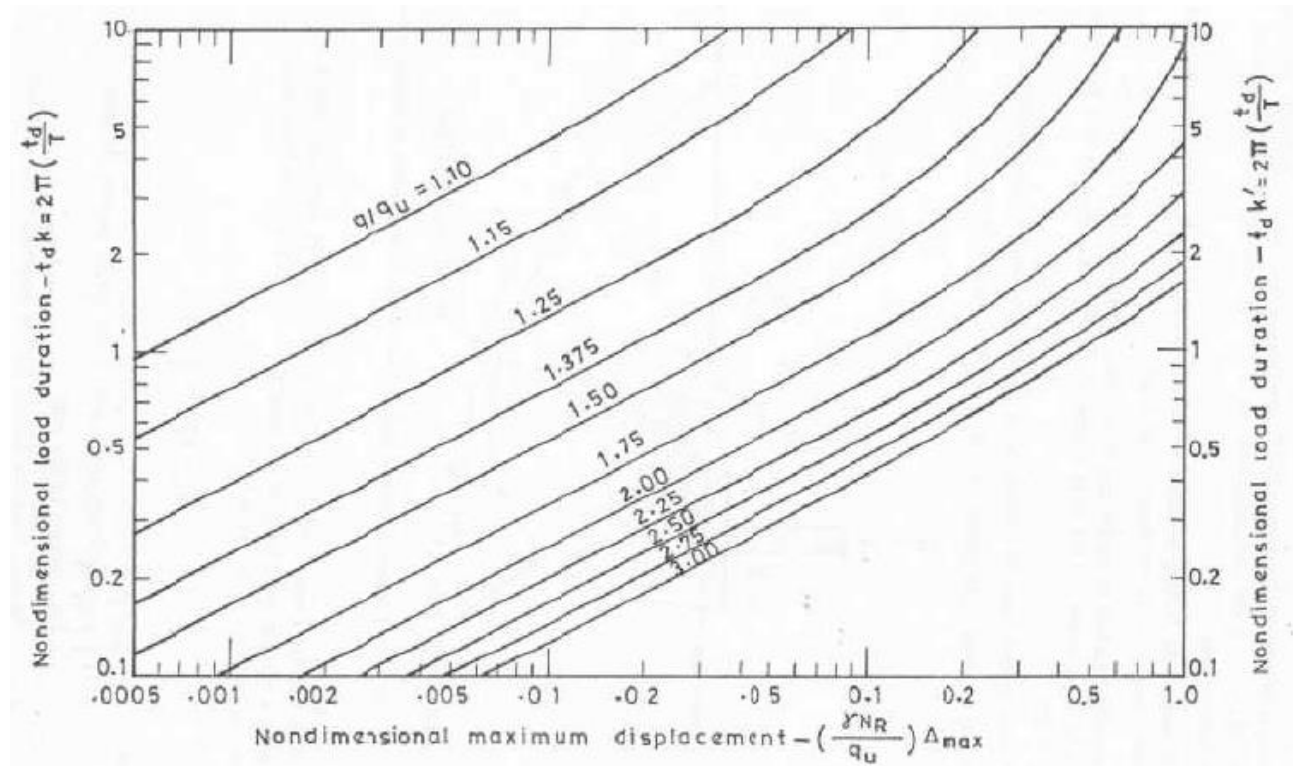
$$K' = \sqrt{\frac{2N_R}{N_I B}} = 5.8303 * \sqrt{\frac{2}{2.5}} = 5.22 \text{ (s}^{-1}\text{)} \quad \text{and} \quad \frac{q_{d(max)}}{q_u} = \frac{3000}{2204} = 1.36$$

$$t_d K' = 0.3 \times 5.22 = 1.566$$

$$\text{For } \frac{q_{d(max)}}{q_u} = 1.36 \text{ and } t_d K' = 1.566$$

$$\frac{\gamma N_R}{q_u} \Delta_{max} = 0.034, \quad \text{Therefore, } \Delta_{max} = \frac{0.034 * 2204}{18 * 10.0081} = 0.416m$$

Numerical Example -2 (Continued...)



Numerical Example-3

A 2.5 m wide continuous surface footing is subjected to a horizontal transient load of duration 0.4 s applied at a height of 4.0 m from the base of footing. The properties of the soil are $\gamma = 17 \text{ kN/m}^3$, $c = 30 \text{ kN/m}^2$ and $\phi = 32^\circ$. Determine the value of the maximum horizontal load that can be applied on the footing. Also compute the rotation at time t equal to 0.6 s.

Solution

- (i) Determine Q using a suitable factor of safety ($= 2.0$). $c = 30 \text{ kN/m}^2$, $\phi = 32^\circ$, $\gamma = 17 \text{ kN/m}^3$ and $B = 2.5 \text{ m}$

$$N_c = \frac{e^{2\pi \tan \phi} - 1}{\tan \phi} = \frac{e^{2\pi \tan 32} - 1}{\tan 32} = 79.4$$

$$N_\gamma = \frac{4 \tan \phi (e^{3\pi \tan \phi} + 1)}{9 \tan^2 \phi + 1} = \frac{4 \tan 32 (e^{3\pi \tan 32} + 1)}{9 \tan^2 32 + 1} = 200$$

Numerical Example-3

$$Q = \frac{B}{F} (cN_c + \frac{1}{2} \gamma B N_\gamma) = \frac{2.5}{2} (30 * 79.4 + \frac{1}{2} 17 * 2.5 * 200) = 8290 \text{ kN}$$

(ii) Determine ψ , ϵ , μ_c β and $\sin(\eta)$

$$H = 4.0 \text{ m}, t_d = 0.4 \text{ s}$$

$$\psi = \frac{(e^{2\pi \tan \phi} - 1)}{2 \tan \phi} = \frac{(e^{2\pi \tan 32} - 1)}{2 \tan 32} = 39.7$$

$$\epsilon = \frac{\tan \phi (e^{3\pi \tan \phi} + 1)}{9 \tan^2 \phi + 1} = \frac{\tan 32 (e^{3\pi \tan 32} + 1)}{9 \tan^2 32 + 1} = 50$$

$$\mu_c = \frac{(e^{4\pi \tan \phi} - 1)}{16 \tan \phi} = \frac{(e^{4\pi \tan 32} - 1)}{16 \tan 32} = 256$$

$$\beta = \frac{(e^{3\pi \tan \phi} + 1)}{3(\sqrt{9 \tan^2 \phi} + 1)} = \frac{(e^{3\pi \tan 32} + 1)}{3(\sqrt{9 \tan^2 32} + 1)} = 56.6$$

Numerical Example-3

$$\bar{x} = \frac{-4B \tan^2 \phi (e^{3\pi \tan \phi} + 1)}{(9 \tan^2 \phi + 1)(e^{2\pi \tan \phi} - 1)} = \frac{-4B \tan^2 32 (e^{3\pi \tan 32} + 1)}{(9 \tan^2 32 + 1)(e^{2\pi \tan 32} - 1)} = -2.52B$$

$$\bar{z} = \frac{4B \tan \phi (e^{3\pi \tan \phi} + 1)}{3(\sqrt{9 \tan^2 \phi + 1})(e^{2\pi \tan \phi} - 1)} = \frac{4B \tan 32 (e^{3\pi \tan 32} + 1)}{3(\sqrt{9 \tan^2 32 + 1})(e^{2\pi \tan 32} - 1)} = 2.85B$$

$$\sin \eta = \frac{\bar{z}}{\sqrt{(\bar{x}^2 + \bar{z}^2)}} = \frac{2.85B}{\sqrt{((-2.52B)^2 + (2.85B)^2)}} = 0.75$$

(iii) Determine k , A , and E

$$k = \sqrt{\frac{g\beta \sin \eta}{\mu_c B}} = \sqrt{\frac{9.81 * 56.6 * 0.75}{256 * 2.5}} = 0.807$$

$$A = \frac{g}{(\gamma B^4 \mu_c)} = \frac{9.81}{(17 * 2.5^4 * 256)} = 0.0000577$$

$$E = \psi c B^2 + \gamma B^3 = 39.7 * 30 * 2.5^2 + 50 * 17 * 2.5^3 = 20700 \text{ kN}$$

Numerical Example-3

(iv) Determine $M_{d(max)}$ in term of λ

$$M_{d(max)} = H^* Q_{d(max)} = H^* \lambda * Q = 4 * 8290 * \lambda = 33160\lambda$$

(v) Determine λ_{cr} which corresponds to $\alpha = 0$

$$\text{For } t = t_d = 0.4, \alpha = \frac{A}{k^2} \left(E - \frac{1}{2} QB \right) \cos(kt) - \frac{A}{k^3} \cdot \frac{M_{d(max)}}{t_d} \sin(kt) + \frac{A}{k^2} \left(\frac{M_{d(max)}t}{t_d} + \frac{1}{2} QB - E \right)$$

$$\alpha = \frac{0.0000577}{0.807^2} \left(20700 - \frac{1}{2} 8290 * 2.5 \right) \cos(0.8070 * 0.4) - \frac{0.0000577}{0.807^3} * \frac{33160\lambda}{0.40} \sin(0.8070 * 0.4) + \frac{0.0000577}{0.807^2} \left(\frac{33160\lambda * 0.4}{0.40} + \frac{1}{2} 8290 * 2.5 - 20700 \right)$$

$$\alpha = 0.050\lambda - 0.0474$$

$$\text{For } \alpha = 0, \lambda = 0.948 = \lambda_{cr}$$

Numerical Example-3

(vi) Determine $M_{d(max)}$ for $\lambda = \lambda_{cr}$

$$M_{d(max)} = 33160 \lambda_{cr} = 33160 \times 0.948 = 31436 \text{ kN-m}$$

(vii) Determine G_1 and G_2

$$\begin{aligned} G_1 &= \frac{A}{k^2} \left(E - \frac{1}{2} QB \right) \cos(kt_d) - \frac{A}{k^3} \cdot \frac{M_{d(max)}}{t_d} \sin(kt_d) + \left(\frac{AM_{d(max)}}{k^2} \right) \\ &= 0.8685 - 2.89 \times 0.948 + 2.94 \times 0.948 = 0.9159 \end{aligned}$$

$$\begin{aligned} G_2 &= -\frac{A}{k} \left(E - \frac{1}{2} QB \right) \sin(kt_d) - \frac{A}{k^2} \cdot \frac{M_{d(max)}}{t_d} \cos(kt_d) + \left(\frac{AM_{d(max)}}{k^2 t_d} \right) \\ &= -0.9159 \times 0.807 \sin(0.3228) - 9.1 \times 0.948 \times 0.807 \cos(0.3228) + 2.94 \times 0.948 \\ &= -4.05 \end{aligned}$$

Numerical Example-3

(viii) Determine α for $t = 0.6$ s

$$\begin{aligned}\alpha &= \frac{1}{k} [G_1 k \cos(kt_d) - G_2 k \sin(kt_d)] \cos(kt) + \\ &\quad \frac{1}{k} [G_1 k \sin(kt_d) - G_2 k \cos(kt_d)] \sin(kt) + \frac{A}{k^2} \left(\frac{1}{2} QB - E \right) \\ &= \frac{1}{0.807} [0.9159 \times 0.807 * \cos(0.807 \times 0.4) + 4.05 \times 0.807 * \sin(0.807 \times 0.4)] \cos(0.807 \times 0.6) \\ &\quad + \frac{1}{0.807} [0.9159 \times 0.807 * \sin(0.807 \times 0.4) + 4.05 \times 0.807 * \cos(0.807 \times 0.4)] \sin(0.807 \times 0.6) \\ &\quad + \frac{0.0000577}{0.807^2} * \left[\frac{1}{2} \times 8290 \times 2.5 - 20700 \right] \\ &= -0.8229 \text{ rad}\end{aligned}$$

References

1. **Prakash S. (1981), "Soil Dynamics", McGraw-Hill Company, New York**
2. **Saran S. (2006), "Soil Dynamics & Machine Foundation", Galgotia Pub. Pvt. Ltd, New Delhi.**

Thank You

